On the movement of ships in restricted waterways

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In this paper some simple theoretical considerations concerning the movement of ships in restricted waterways are discussed, and it is shown that for a ship towed from the bank, or by any external force, there are three distinct speed ranges: subcritical, critical and supercritical. In the subcritical and supercritical ranges, Bernoulli's equation and the continuity equation are satisfied everywhere by a state of steady motion relative to the ship, but in the critical range these laws require that a quantity of fluid is piled up continuously ahead of the ship in the form of a bore. Experimental confirmation is given by means of photographs of model tests.

1. Introduction

When a ship passes along a canal or other restricted waterway, it has been observed that the distance between the keel of the ship and the canal bottom decreases as the speed increases, and in fact on occasions the ship has been known to strike the bottom. This phenomenon is known as 'squatting'. In response to a request from the Manchester Ship Canal Company, the author undertook an investigation into the problem of squatting. In addition to a theoretical consideration of the problem many model tests were carried out in the Whitworth Laboratories at Manchester University.

One of the main outcomes of the investigation was the elucidation of a distinctive yet little-known general property concerning the movement of floating bodies along canals. This is that there are three types of flow régime according to different speed ranges. The principal aim of the present paper is to explain this property and to show, for each of the three cases, the relationships between the salient parameters of the problem, such as the dimensionless ship speed (i.e. a Froude number) and a dimensionless parameter measuring the degree of squatting. The theoretical model considered is severely simplified, but photographs of an experimental demonstration are presented which confirm the essential distinctions between the three possible régimes.

2. Theoretical treatment

To reduce the problem to its simplest form, the following assumptions are made:

(1) The ship moves with a constant velocity V_1 along a canal of uniform rectangular cross-section breadth b and undisturbed depth y_1.

(2) The canal extends rectilinearly to infinity in both directions.

(3) The cross-section of the ship is uniform over its whole length and the end effects are ignored.

(4) The velocities of the water particles in any cross-section are constant over that cross-section.

(5) The loss of head due to friction is neglected.

(6) The effects of the secondary wave system are ignored.

(7) Any force necessary to move the ship is provided externally—not by means of a propeller which causes a reaction on the water.

Let us choose a frame of reference moving with the ship, so that the problem is reduced to one of steady flow. The sides and bottom of the canal must of course be considered to be moving past the ship with a velocity V_1; but since we are in effect dealing with an ideal fluid, this motion of the wetted boundary will not influence the flow.

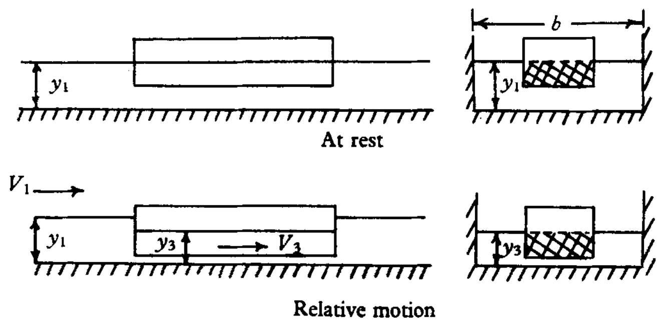

FIGURE 1. Idealized motion of the ship in the subcritical range.

With reference to figure 1, we have as the equation continuity

$$V_1 b y_1 = V_3(b y_3 - A), \tag{1}$$

where A is the underwater cross-sectional area of the ship, y_3 the depth of the water alongside the ship, and V_3 the velocity of the water alongside the ship. Bernoulli's equation takes the form

$$\frac{V_1^2}{2g} + y_1 = \frac{V_3^2}{2g} + y_3. \tag{2}$$

Combining equations (1) and (2), we get

$$V_1 b y_1 = V_3 \left\{ b y_1 - \frac{(V_3^2 - V_1^2)\, b}{2g} - A \right\}. \tag{3}$$

Introducing the dimensionless parameters

$$S = \text{blockage factor} = \frac{A}{b y_1},$$

$$d = \frac{y_1 - y_3}{y_1}, \quad F_1 = \frac{V_1}{(g y_1)^{\frac{1}{2}}}$$

it can be shown that (3) reduces to

$$F_1 = \left[\frac{2d(1-d-S)^2}{1-(1-d-S)^2} \right]^{\frac{1}{2}}. \tag{4}$$

If the relationship between F_1 and d for various values of the blockage factor S is plotted, a remarkable property comes to light (see figure 2). It would appear that for a given blockage factor there is a certain speed beyond which it is impossible for the ship to go. At this 'maximum' speed the ship will have squatted by a certain amount shown by the value of d corresponding to the maximum value of F_1. If the value of x/y_1 (where x is the clearance between the bottom of the ship and the bottom of the canal when stationary) is less than this limiting value of d, then the ship will strike the canal bottom at a speed corresponding to $d = x/y_1$.

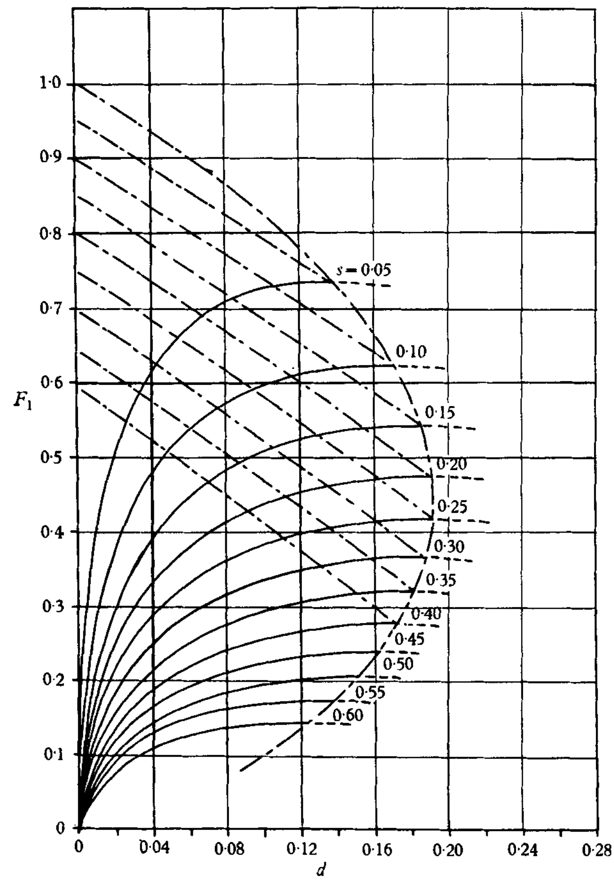

FIGURE 2. The relationship between the Froude number F_1 (based on the speed of the ship and the undisturbed depth of the canal) and the dimensionless 'squat' d of the ship for various values of the blockage factor S.

This speed of course will be less than the maximum for that one blockage factor. If, however, the value of x/y_1 is greater than the limiting value of d, it is necessary to ask what happens to the ship if it attempts to exceed the 'maximum' speed?

3. Conditions at higher speeds

At first sight a possible solution to this problem is that the ship would catastrophically sink if the velocity was increased by a fraction; but a more likely solution follows from an idea given in a publication by Lap (1950) in which he considers the work of Krietner (1934).

From equation (3) we can derive

$$\frac{V_1}{(gy_1)^{\frac{1}{2}}} = \frac{V_3}{(gy_1)^{\frac{1}{2}}}\left[1 - S - \frac{1}{2}\left\{\frac{V_3^2}{gy_1} - \frac{V_1^2}{gy_1}\right\}\right].$$

If now $V_3/(gy_1)^{\frac{1}{2}}$ is plotted against $V_1/(gy_1)^{\frac{1}{2}}$ for a range of values of S, figure 3 is obtained. Here it can be seen that, as V_1 increases, so does V_3 up to a certain maximum value beyond which once again there is apparently no real solution.

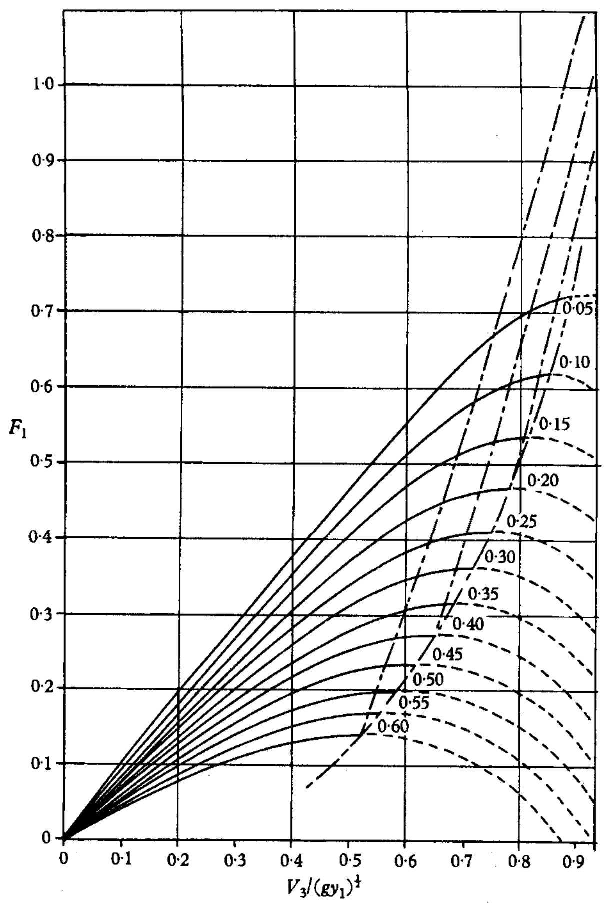

FIGURE 3. The relationship between the Froude numbers based on the speed of the ship and the relative velocity alongside the ship for various values of the blockage factor S.

In other words, up to this maximum value of V_3 the quantity of water flowing past the sides of the ship is capable of adjusting itself to take all the water presented by the corresponding value of V_1. Beyond the maximum, however, V_3 is no longer capable of keeping pace with V_1, and the excess water must be piled up in front of the ship.

Before this problem can be considered further, it is necessary to investigate what happens fore and aft of the ship as the water is piled up.

4. Idealized 'piston' motion in a canal

Consider a canal of uniform rectangular cross-section (see figure 4) extending rectilinearly to infinity in both directions. The canal is filled with a perfect liquid to a depth of y_1. Across the cross-section, and completely blocking it, is a flat plate capable of being moved along the length of the canal at a constant speed. If in

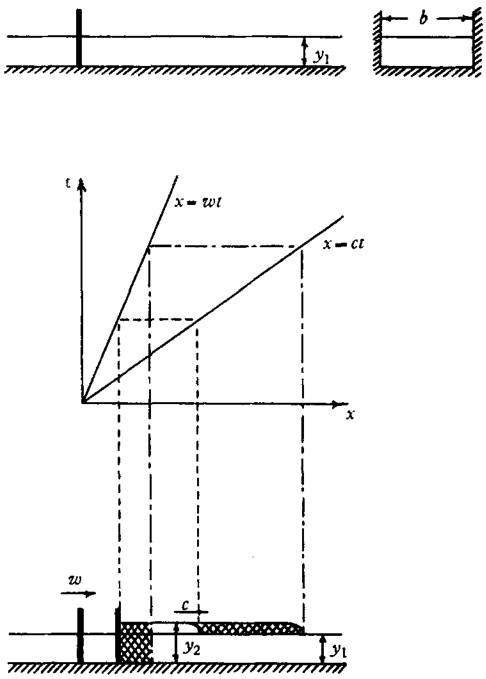

FIGURE 4. Idealized piston motion in a canal.

fact the plate is moved with a constant velocity w, the resulting motion ahead of the plate will be as shown and the speed of the bore front will be given by

$$c = \left[g\frac{y_2}{y_1}\left(\frac{y_1+y_2}{2}\right)\right]^{\frac{1}{2}}$$

(cf. Stoker 1957, p. 328).

To investigate the motion at the rear of the plate it is convenient to consider first the motion of a simple wave in shallow water (Stoker 1957, Ch. 10). If u the horizontal velocity component and c the propagation speed are both functions of x, the horizontal distance travelled, and of time t, it can be shown that in the (x, t)-plane we have two sets of characteristics C_1 and C_2 which are the solution curves of the ordinary differential equations

$$C_1: \frac{dx}{dt} = u+c,$$

$$C_2: \frac{dx}{dt} = u-c.$$

Also

$$u+2c = k_1 \quad \text{a constant along } C_1,$$
$$u-2c = k_2 \quad \text{a constant along } C_2.$$

This can now be used to determine the form of a disturbance behind a plate which is completely blocking the canal, and which is moving at a constant velocity w in still fluid of constant depth y_1. The acceleration of the plate to the velocity w may be either gradual or instantaneous but provided the position of the plate is known for all t, solutions are possible.

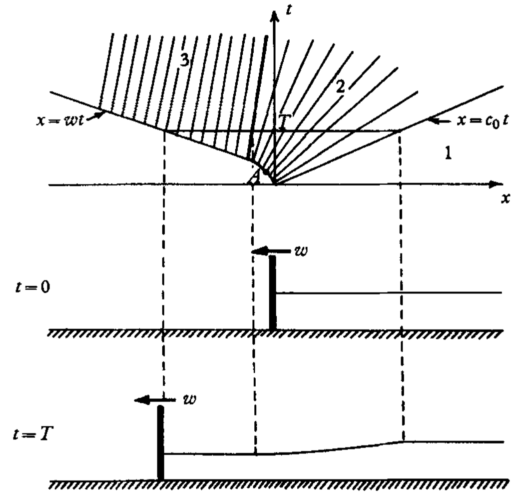

FIGURE 5

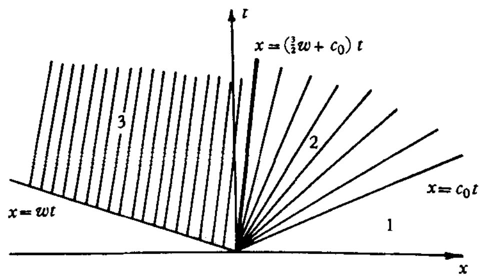

FIGURE 6

FIGURE 5. The (x, t)-relationship describing the disturbance to the rear of a plate which completely blocks the canal cross-section and accelerates gradually from rest to a constant velocity w.

FIGURE 6. The (x, t)-relationship describing the disturbance to the rear of a plate which completely blocks the canal cross-section and accelerates instantaneously to a constant velocity w.

The situation when the acceleration of the plate is not instantaneous is given in figure 5. Zone 1 represents the quiet area where the disturbance has not reached, and zone 2 represents a region of variable depth joining zone 1 to zone 3 which is another zone of constant depth. The two equations which define the straight characteristics in zone 2 are determined as follows:

$$u_0 - 2c_0 = u_a - 2c_a = -2c_0, \quad c_a = \tfrac{1}{2}u_a + c_0,$$
$$\frac{dx}{dt} = u_a + c_a = \tfrac{3}{2}u_a + c_0.$$

Since $c_a = [g(y_1 + \eta_a)]^{\frac{1}{2}}$, the surface elevation η_a can be determined anywhere in zone 2 at any time t.

Similarly the equation for zone 2 when the acceleration of the plate is assumed to be infinite (figure 6) is derived as follows:

$$\frac{dx}{dt} = \frac{x}{t} = \tfrac{3}{2}u + c_0, \quad u = \frac{2}{3}\left(\frac{x}{t} - c_0\right),$$
$$c = \tfrac{1}{2}u + c_0 = \tfrac{1}{3}(\tfrac{1}{2}x + 2c_0).$$

Again knowing $c = [g(\eta + y_1)]^{\frac{1}{2}}$, the surface elevation η can be determined anywhere in zone 2 at any time t.

Thus for the given condition we can accurately describe the motion fore and aft of the plate when it moves with a uniform velocity w.

5. The critical and supercritical ranges

Whilst on the face of it there seems to be little connexion between a plate which completely blocks the canal and a ship which is floating and in addition only blocking a fraction of the canal cross-section, the information just derived can in fact be used. The water piled up in front of the ship will cause, as in the case of the plate, a surge or bore to travel ahead and consequently a negative disturbance aft of the ship. This is illustrated in figure 7. It should be noted that we are still assuming that there are no end effects and the ship therefore retains an even keel.

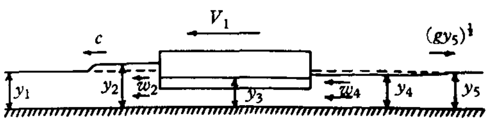

FIGURE 7. Idealized motion of the ship in the critical range.

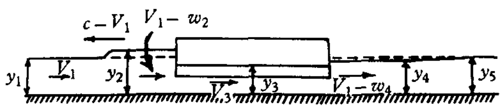

FIGURE 8. Idealized motion of the ship in the critical range, the frame of reference moving with the ship.

It is easily seen that the expression for c has the same form as before. Considering now the motion to the aft of the ship, we have, for the total quantity passing the ship,

$$b(V_1 - w_2)\, y_2 = b(V_1 - w_4)\, y_4.$$

But we know that

$$c = (gy_4)^{\frac{1}{2}} = (gy_5)^{\frac{1}{2}} - \tfrac{1}{2}w_4 = (gy_1)^{\frac{1}{2}} - \tfrac{1}{2}w_4,$$

$$w_4 = 2[(gy_1)^{\frac{1}{2}} - (gy_4)^{\frac{1}{2}}],$$

$$(V_1 - w_2)\, y_2 = [V_1 - 2\{(gy_1)^{\frac{1}{2}} - (gy_4)^{\frac{1}{2}}\}]\, y_4.$$

Thus, given y_2, we have sufficient information to determine y_4 and w_4. If, therefore, a relationship between y_2 and V_1 can be determined, we have a complete picture of the behaviour of the ship under the ideal conditions assumed.

When the ship does start to push forward a quantity of water, the depth and velocity immediately in front of the ship will alter and V_3 (see figure 8) will adjust itself to pass the maximum under the new conditions. With sufficient computational labour it is possible to work out the values of $V_3/(gy_1)^{\frac{1}{2}}$ and d for values of $V_1/(gy_1)^{\frac{1}{2}}$ in excess of the critical, i.e. when the steady state ceases to exist. These results are shown as chain lines in the graphical representation in figures 2 and 3. As $V_1/(gy_1)^{\frac{1}{2}}$ increases, $V_3/(gy_1)^{\frac{1}{2}}$ continues to increase but at a much slower rate than before. At the same time d decreases in value, i.e. the ship begins to rise.

But what happens as $V_1/(gy_1)^{\frac{1}{2}}$ becomes equal to and exceeds unity? If equations (1) and (2) are re-examined it will be seen that in addition to the solution already plotted for the subcritical range there are solutions for $F_1 > 1$. These additional solutions are shown for a range of blockage factors in figures (9) and (10).

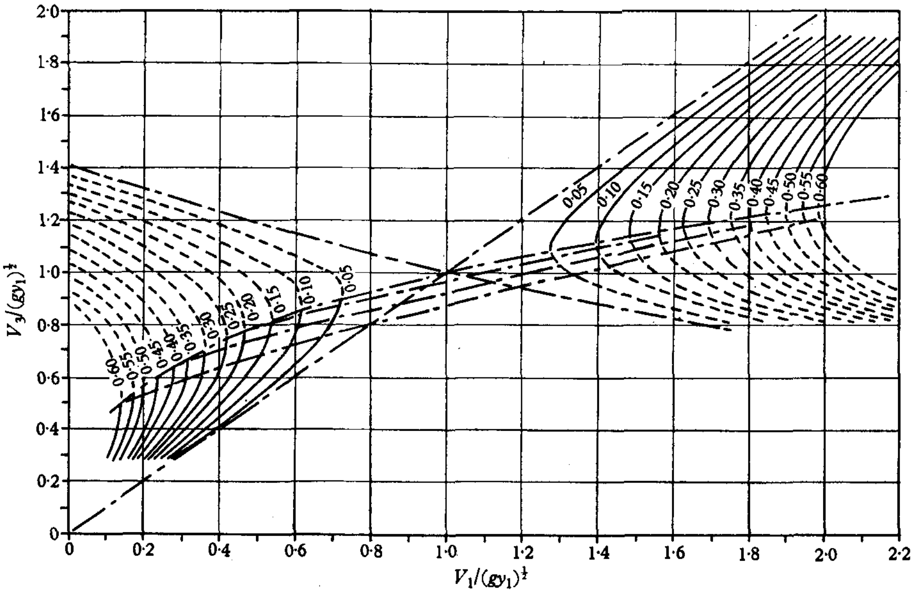

FIGURE 9. The relationship between the Froude numbers based on the speed of the ship and the relative velocity alongside the ship for various values of the blockage factor S.

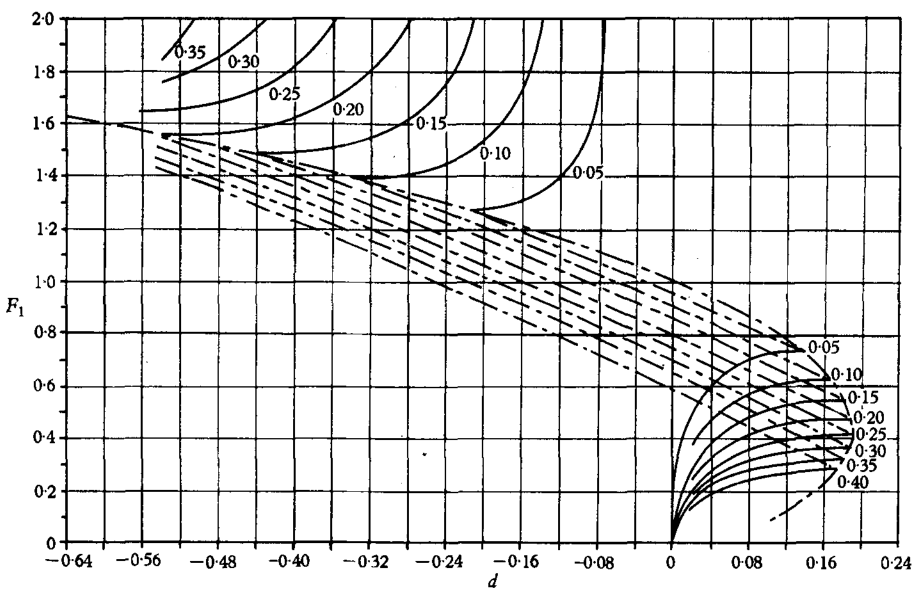

FIGURE 10. The relationship between the Froude number F_1 (based on the speed of the ship and the undisturbed depth of the canal) and the dimensionless 'squat' of the ship for various values of the blockage factor S.

It is clear from a study of these graphs that for a ship moving in a restricted passage of fluid (or indeed for any floating body which is moving in a restricted passage of fluid, or which is stationary in a moving body of fluid), there are three distinct speed ranges: subcritical, critical and supercritical. In the subcritical range the Bernoulli and continuity equations are satisfied by a steady state everywhere, the excess velocity alongside the ship causing a decrease in the depth of fluid and consequently the squatting of the ship. In the supercritical range the 'steady-state' Bernoulli and continuity equations are again satisfied everywhere, but, contrary to the subcritical range, there is a reduced velocity alongside the

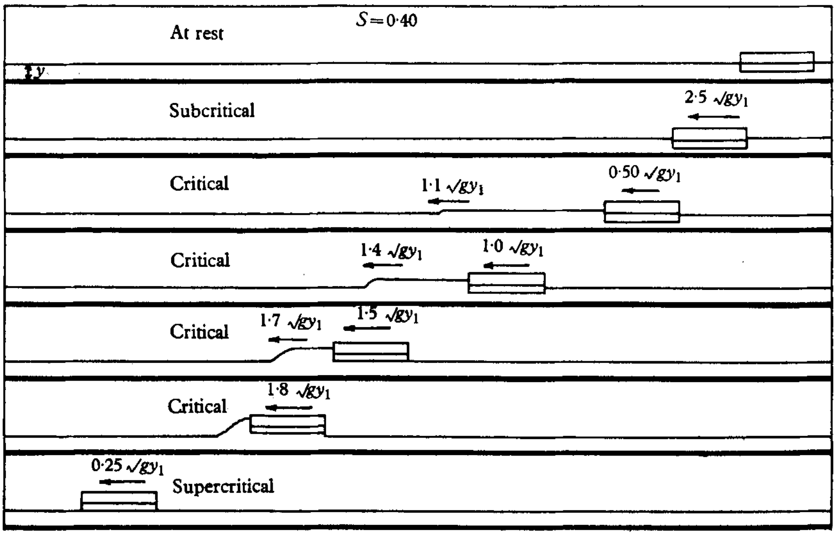

FIGURE 11. The idealized behaviour of a ship as it moves at speeds in the subcritical, critical and supercritical range. For ease of comparison the diagrams give the position of the ship and bore at one fixed time after starting instantaneously from rest.

ship and hence an increase in the depth of fluid causing the ship to rise above its original static position. Connecting the subcritical and the supercritical is the critical range where the Bernoulli and continuity equations can only be satisfied in the vicinity of the ship if a quantity of fluid is piled up ahead of it. This causes a bore to advance ahead and a negative disturbance to move aft and away from the ship. Contrary to the other two ranges in which the motion is steady, the motion in this case is unsteady.

It can be shown that as F_1 increases above the lower critical speed the height of the bore increases. At the same time the absolute speed of the bore front increases but its speed relative to the ship decreases, i.e. the ship tries to catch up the bore front. At the upper critical speed the bore height is a maximum and the ship is travelling at a speed only slightly less than that of the bore front, a small quantity of fluid still being pushed ahead. Immediately the upper critical speed is exceeded however the ship 'jumps' on the bore and the bore as such disappears, the ship no longer pushing ahead any fluid. Figure 11 shows quite clearly the effect as F_1 increases through the subcritical and supercritical range for the ideal case considered.

Clearly the shape of the ship and the viscous effects of the fluid will affect the results just derived. These effects along with the case of self-propulsion are to be dealt with in a separate paper.

This phenomenon was conclusively demonstrated in the laboratory by towing a small ship (8 in. long) along a small Perspex channel filled with water. The photographs given in figures 12 (*a*), (*b*) and (*c*) (plate 1) show the ship in the sub-critical, critical and supercritical ranges, respectively. It will be noticed that in contrast to the ideal case considered the ship takes on a considerable trim in the critical range.

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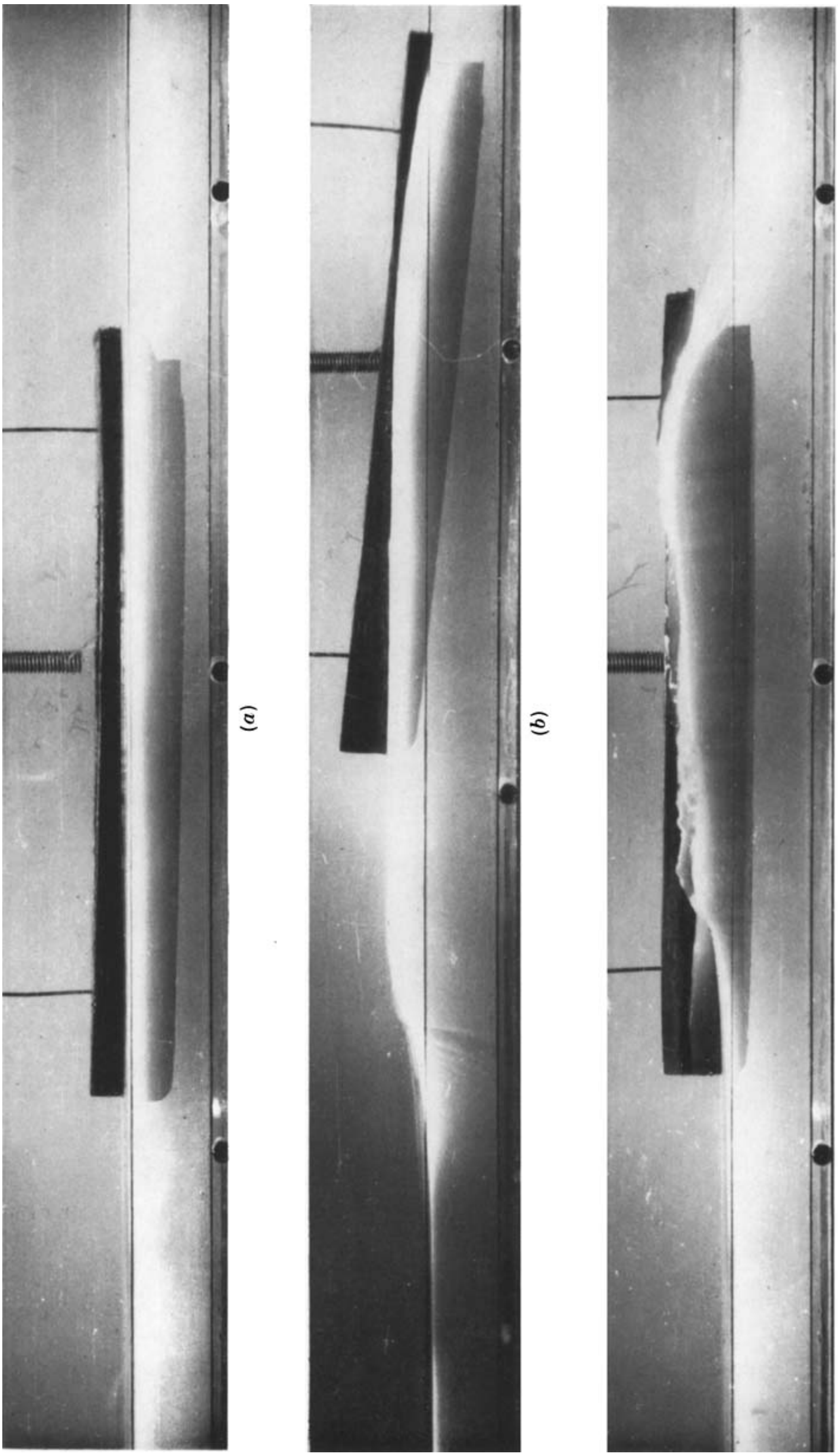

FIGURE 12. (*a*) Subcritical range. (*b*) Critical range. The ship has only just started to move and therefore the distance between the bow of the ship and the bore front is small. (*c*) Supercritical range.